

Could we observe the discreteness of Quantum-Gravity length and area operators ?¹

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Abstract

Several proposals for Quantum Gravity involve length and area operators with discrete eigenvalues. I show that the analyses of some simple procedures for the measurement of areas and lengths suggest that this discreteness characterizing the formalism might not be observable. I also discuss a possible relation with the so-called κ deformations of Poincaré symmetries.

The most fascinating problem that confronts the physics community is the one of reconciling Quantum Mechanics with Gravity. The problem can be discussed using very familiar terms and we even have reasonably solid (although not conclusive [1, 2]) arguments to infer that the Planck length, which is nothing more than a combination of familiar scales, should characterize the onset of observably large “Quantum Gravity” phenomena. These familiar aspects of the problem have somehow rendered even more frustrating the lack of success of the decades of efforts devoted to Quantum Gravity. However, one should not forget that, although the problem can be discussed in familiar terms, the “jump” required in order to reach the Planck length ($L_P \sim 10^{-33}cm$) from presently-available experimental information (which can be roughly characterized by the mass of the gauge bosons mediating the weak interactions, *i.e.* of order $M_{W,Z}^{-1} \sim 10^{-16}cm$) is gigantic even by comparison with previous major “revolutions” in our description of the physical world. Trying to deduce the nature of Planck-length physics from the experimental information we have presently available is an extremely

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ambitious task, so much so that some of our colleagues feel it would be futile. While I shall not endorse this *a priori* judgement of futility, it is worth noticing that the comparison of the relevant scales (sharpness of the probes versus characteristic scale of the physics being studied) indicates that one would be in better shape trying to discover and describe the weak interactions only using the information that can be gotten sitting in the stands of a stadium by watching a sporting event!

These observations should at least act as a warning not to trust too much the intuition coming from presently-accessible physical regimes in trying to obtain a description of the Planck-length regime. In particular, the successes of Quantum Mechanics provide no guarantee that all of its postulates should find use in the description of very short distances. Within the Quantum-Gravity community it has been extensively debated whether ordinary Quantum Mechanics could be the appropriate theoretical framework for Quantum Gravity. Some authors feel that we have convincing evidence (in the nature of the “tension” between Quantum Mechanics and Gravity) in support of the fact that Quantum Mechanics should be modified in order to accommodate Gravity while other authors feel that the evidence presently available is not solid enough to justify this conclusion. However, in light of the remarkable “jump” emphasized above it seems that, even if we all agreed that at present we cannot assume that Quantum Mechanics must be violated, we would still have to allow for the possibility that such violations might emerge at some point down the road to the Planck scale. The problem becomes then the one of getting some intuition for the nature of plausible candidates for such violations, considering that presently attainable experimental data can be of very little help. It is in this respect that some recent analyses of gedanken experiments can be most useful. A final test of the results of those analyses must wait for the correct Quantum Gravity or at least some relevant experimental data, but those results already contribute to the general development of Quantum-Gravity research by guiding us toward alternative scenarios which would otherwise not be considered.

In the present paper I discuss one of the ways in which Quantum-Gravity physics could violate some postulates of Quantum Mechanics. Specifically, I shall be concerned with two properties of ordinary Quantum Mechanics: (1) that it allows a well-defined (although formal) limit in which the devices used in measurements behave “classically”, in the sense that their positions are completely under the control of the observer, and (2) that any given observable can be measured with total accuracy (at the price of renouncing any information on a conjugate observable) in the limit in which the devices composing the measuring apparatus behave classically. In particular, the property (2) would in principle allow to uncover any discreteness in the spectrum of a quantum operator. This is quite important since many Quantum-Gravity scenarios involve length, area and volume operators which have discrete eigenvalues. For example, an area operator [3] with discrete eigenvalues is one of the most intriguing aspects of Canonical/Loop Quantum Gravity [4, 5, 6], which is a Quantum-Gravity approach that (while being like all its competitors only at the early stages of development and missing any

experimental support) has passed quite a few tests of formal and conceptual consistency.² Even more popular is the expectation that the length operator should have discrete eigenvalues, at least in some Quantum-Gravity scenarios.

In this paper, I shall assume that indeed such discrete length and area operators correctly describe physical lengths and physical areas in the Quantum-Gravity formalism, and investigate how this discreteness that characterizes the formalism would affect the outcome of experiments. Some of the points made in the following are relevant for the study of any diffeomorphism-invariant length and area operator (whether or not the spectrum is discrete). Other aspects of the analysis apply only to diffeomorphism-invariant operators with discrete spectrum, but still the details of the spectrum are never important for the line of argument here proposed. For simplicity, the reader can assume that the length operator has eigenvalues \mathcal{L}_n given by integer multiples of the Planck length

$$\mathcal{L}_n = nL_P , \quad (1)$$

which is the type of quantization most commonly considered, while the area operator has eigenvalues \mathcal{A}_n given by half-integer multiples of the square of L_P :

$$\mathcal{A}_n = \frac{n}{2}L_P^2 , \quad (2)$$

which is the type of quantization found [7] in the Husain-Kuchař-Rovelli model.

Let us start by considering the measurability of the distance L between (the centers of mass of) two bodies. In Ref. [11] the measurement of the distance L is discussed in terms of the Wigner measurement procedure [12, 13], which relies on the exchange of a light signal between the two bodies. The setup of the measuring apparatus schematically requires *attaching* a light-gun, a clock, and a detector to one of the bodies and *attaching* a mirror to the other body. By measuring the time T needed by the signal for a two-way journey between the bodies one also obtains a measurement of L . Within this setup it is easy to realize that δL can vanish only if all devices used in the measurement behave classically. One can consider for example the contribution to δL coming from the uncertainties that affect the relative motion of the clock with respect to the center of mass of the system composed by the light-gun and the detector. It is easy to show [11, 12, 13] that

$$\delta L \geq \sqrt{\frac{(M_c + M_{l+d})\hbar T}{2M_c M_{l+d}}} , \quad (3)$$

²Although in the context considered in Ref. [3] areas are not diffeomorphism-invariant, the analysis reported by Rovelli in Ref. [7] suggests that a discrete spectrum should also characterize areas specified in a diffeomorphism-invariant manner [8, 9] by matter fields. In fact, in Ref. [7] this discreteness was analyzed within the model obtained by introducing matter fields in the Husain-Kuchař Quantum-Gravity toy model [10], whose area operator is completely analogous to the one of Canonical/Loop Quantum Gravity.

where M_c is the mass of the clock and M_{l+d} is the total mass of the system composed of the light-gun and the detector. Clearly, Eq. (3) implies that $\delta L = 0$ can only be achieved in the “classical-device limit,” understood as the limit of infinitely large M_c and M_{l+d} . This is consistent with the nature of the ordinary Quantum-Mechanics framework, which relies on classical devices. However, once gravitational interactions are taken into account the classical-device limit is no longer available. Large values of the masses M_c and M_{l+d} necessarily lead to great distortions of the geometry, and well before the $M_c, M_{l+d} \rightarrow \infty$ limit the Wigner measurement procedure can no longer be completed. Since the classical limit $M_c, M_{l+d} \rightarrow \infty$ is not available, from Eq.(3) one concludes that in Quantum Gravity the uncertainty on the measurement of a length grows with the time T required by the measurement procedure (as it happens in presence of decoherence effects [14]). In fact, Eq.(3) can motivate [11] the expectation for a minimum uncertainty for the measurement of a distance L of the type

$$\text{minimum} [\delta L] \sim \sqrt{cTL_{QG}} \sim \sqrt{LL_{QG}} , \quad (4)$$

where L_{QG} is a Quantum-Gravity length scale that characterizes the mentioned limitations due to the absence of classical devices, and the relation on the right-hand side follows from the fact that T is naturally proportional [11, 13] to L . Although L_{QG} emerges in a way that does not appear to be directly related to the Planck length, it seems plausible [11] that $L_{QG} \sim L_P$.

In the following I shall assume that indeed (4) holds.

Let us now consider the measurement of areas. I shall consider the measurement procedure proposed by Rovelli in Ref. [7]. There, for simplicity, the matter fields that specify the surface whose area is being measured are taken to form a metal plate. The area \mathcal{A} of this metal plate is measured using an electromagnetic device that keeps a second metal plate at a small distance d and measures the capacity C of the so formed capacitor. Of course, measuring d and C , and assuming that $d \ll \sqrt{\mathcal{A}}$, one also measures \mathcal{A} as

$$\mathcal{A} = Cd , \quad (5)$$

where I chose for simplicity units in which the relevant permittivity is 1.

According to Eq. (5), in general the uncertainty in the measurement of the area \mathcal{A} receives contributions from the uncertainties in the determination of C and d . Since I am aiming for a final result formulated as a measurability bound (*i.e.* a lower bound on the uncertainty), it is legitimate to ignore the contribution coming from the uncertainty in C and focus on the contribution coming from the uncertainty in d

$$\delta \mathcal{A} \geq C \delta d = \frac{\delta d}{d} \mathcal{A} , \quad (6)$$

where I also used again Eq. (5) to eliminate C .

From the bound (4) on the measurability of distances it follows that $\delta d/d \geq \sqrt{L_{QG}/d}$ and therefore

$$\delta \mathcal{A} \geq \sqrt{L_{QG}} \frac{\mathcal{A}}{\sqrt{d}} . \quad (7)$$

This relation confronts us with a scenario similar to the one of Eq. (3). It formally admits a limit ($d \rightarrow \infty$) in which the area could be measured with complete accuracy, but this limit cannot be reached within the constraints set by the nature of the measurement procedure. In fact, the relation (5), on which the measurement procedure is based, only holds for $d \ll \sqrt{\mathcal{A}}$, and in considering larger and larger d one quickly ends up losing all information on \mathcal{A} . A rather safe lower bound is therefore obtained by imposing $d \leq \sqrt{\mathcal{A}}$ in Eq. (7), which gives

$$\delta \mathcal{A} \geq \sqrt{L_{QG}} \mathcal{A}^{3/4} . \quad (8)$$

Bounds of the type (4) and (8) would require a significant shift in the physical interpretation of quantization relations such as (1) and (2). In fact, assuming $L_{QG} \sim L_P$, Eq. (4) indicates that the measurement of a given length of order nL_P would be affected by an uncertainty of at least $\sim L_P(n)^{1/2}$, *i.e.* (for every length with $n > 1$) an uncertainty much larger than the L_P quanta. Similarly, assuming $L_{QG} \sim L_P$, Eq. (8) indicates that the measurement of a given area of order $nL_P^2/2$ would be affected by an uncertainty of at least $\sim L_P^2(n/2)^{3/4}$, *i.e.* (for every area with $n > 1$) an uncertainty much larger than the $L_P^2/2$ quanta.

Concerning the physical interpretation of Eqs. (4) and (8) one is also naturally led to inquire about the type of symmetries that could result in such structures. Of course, it will be possible to rigorously address this question only once a formalism supporting relations such as (4) and (8) is found; however, some consistency arguments [15, 16] appear to indicate that dimensionful deformations of Poincaré symmetries might be involved. Interestingly, some of the predictions of these deformations of Poincaré symmetries could soon be tested [17] experimentally by exploiting the recent dramatic developments in the phenomenology of gamma-ray bursts [18]. While the interested reader should go to Refs. [15, 16] for a detailed discussion, I shall here just sketch out one of the arguments supporting the connection between dimensionful deformations of Poincaré symmetries and relations such as (4) and (8). I start by observing that the quantum κ -deformed Minkowski space [19, 20]

$$[x_j, x_k] = 0 \quad (9)$$

$$[x_j, t] = \frac{x_j}{\kappa} , \quad (10)$$

can be interpreted as implying that the uncertainties on x_j and t satisfy

$$\delta x_j \delta t \geq \frac{x_j}{|\kappa|} . \quad (11)$$

This uncertainty relation has important implications for Wigner's measurement procedure. In fact, interpreting (11) as a relation between the uncertainty δt in the time when the light probe sets off the clock at the end of its two-way journey and the uncertainty δx (along the axis of propagation of the light probe) in the distance travelled by the probe by that same set-off time, one obtains the relation [16]

$$\delta L \geq \delta x + c \delta t \sim \delta x + \frac{2cL}{|\kappa|\delta x} , \quad (12)$$

which also takes into account that both δx and δt contribute to the total uncertainty in the measurement of L . From (12) one finds that

$$\delta L \geq \sqrt{\frac{2cL}{|\kappa|}} , \quad (13)$$

which reproduces the relation (4) upon appropriate association of the scale κ to the scale L_{QG} .

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